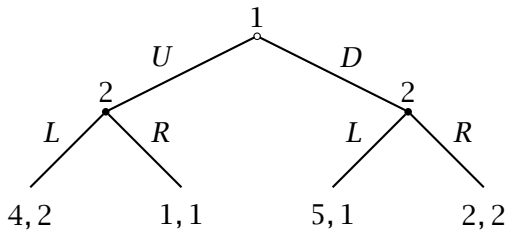
GAMES OF INCOMPLETE INFORMATION

Show all steps and calculations in your answers.

QUESTION 1. Consider the following extensive form game.



(a) Find a subgame perfect equilibrium to this game. Is it unique? Are there any other Nash equilibria?

(b) Suppose that player 2 cannot observe player 1's move. Write down the new extensive form. What is the set of Nash equilibria?

(c) Now suppose that player 2 observes player 1's move correctly with probability $p \in (0, 1)$ and incorrectly with probability $1 - p$. That is, if player 1 chooses U, then player 2 observes U with probability p and observes D with probability $1 - p$. Suppose that this facet of the situation and the value of p are common knowledge. What is the extensive form of this situation? Find all perfect Bayesian equilibria. (Hint: be very careful with the new extensive form; different actions by player 1 will lead to the same information set for player 2; think about what player 2 "sees" and how this relates to player 1's true action. Write out Bayes rule first and derive the best responses. To search for equilibria from these best responses, consider pooling first, then separating, and finally semi-separating equilibria in which both types mix or exactly one type mixes. There are many equilibria in this game.)

QUESTION 2. (REAL MEN DON'T EAT QUICHE.) Consider the game in Figure 1. There are two types of player 1, *real man* and *wimp*, which Nature chooses with probability 0.9 and 0.1, respectively. Both players know this probability but only player 1 observes Nature's move.

Player 1 is sitting in a bar when player 2 walks in. Player 2 is the rowdy type and wants to pick a fight, but he's also a coward, so he only wants to fight a wimp. Player 2 has two

actions, *fight* (F) and *not fight* (N) but before he makes the choice, he observes player 1's behavior.

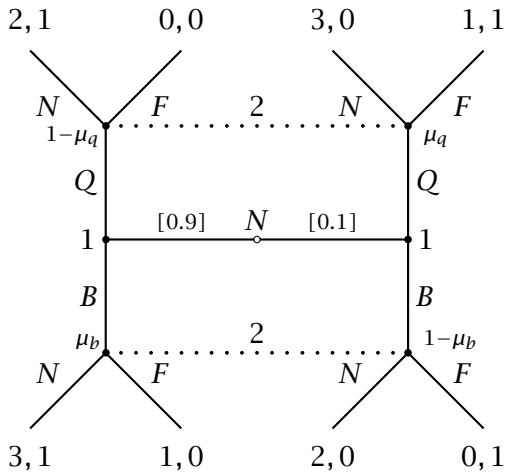


Figure 1: Beer or Quiche.

Player 1 can choose to *drink beer* (B) or *eat quiche* (Q) and obtains one unit of utility if he consumes his most preferred breakfast. Real men prefer beer and wimps prefer quiche. Player 2's payoff does not depend on player 1's breakfast and is 1 if it fights the wimp or if he avoids fighting the real man, and zero otherwise.

Find the perfect Bayesian equilibria of this game. Do any of them appear unreasonable? Why? (Hint: think about the conjectures player 2 must make to support one of the PBE, and then think what this implies for player 1's behavior.)

QUESTION 3. Consider the two strategic-form games in Figure 2. Find the perfect Bayesian Nash equilibria in the following games:

(a) Nature chooses the game to the left with probability $\pi \in (0, 1)$. Neither player observes the choice.

(b) Player 1 observes Nature's choice but player 2 does not.

		Player 2	
		L	R
Player 1	T	$1, 1$	$0, 0$
	B	$0, 0$	$0, 0$

		Player 2	
		L	R
Player 1	T	$0, 0$	$0, 0$
	B	$0, 0$	$2, 2$

Figure 2: Two Strategic-Form Games.

(Hint: construct the strategic forms.)